### Reactive Synthesis for Cyber-Physical System Control

#### Rüdiger Ehlers University of Bremen & DFKI GmbH

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#### Synthesis of reactive systems



# Synthesis of CPS controllers

#### Main Question

How to deal with the **continuous** and **discrete** aspects of the problem at the same time?

#### Approaches

- Abstraction of the continuous world into the purely discrete world + discrete synthesis (e.g., Nilsson et al., 2016)
- Simplifying the continuous parts (Linear hybrid automata / Timed automata) and using a specialized synthesis algorithm for the resulting mixed discrete/continuous model (e.g., Benerecetti et al., 2013; Papusha et al., 2016)
- Continuization" of the discrete parts and using purely continuous methods for controller computation

#### Using a discrete abstraction







#### Using a discrete abstraction











#### Input/Output

#### Input:

- (Sensed) positions of the robots
- Delivery requests

Output:

- Up/Left/Right/Down command of the red robot
- Pickup/drop actions of the red robot



#### Guarantees

- Whenever a button is pressed, then the robot is eventually at the lower left region and performing a pick-up action, while later being in the top right region, performing a drop action, without performing a drop action in between.
- No crashes between the robots



#### Assumptions

- Obstacle can only move in every second step
- Obstacle can only move by one cell per direction per step
- x position of the robot is updated according to its choice
- y position of the robot is updated according to its choice
- No robot jumps further that one cell are possible

#### Assumptions and guarantees in specifications



#### Specification shape

$$(\bigwedge \mathsf{Assumptions}) o (\bigwedge \mathsf{Guarantees})$$

#### Reactive synthesis – Complexity vs. expressivity



#### Reactive synthesis – Complexity vs. expressivity



## Reactive synthesis - Complexity vs. expressivity

#### GR(1) synthesis applications

o ...

- On-chip bus arbiter (Bloem et al., 2007b,a; Godhal et al., 2011)
- High-level robot control (Kress-Gazit et al., 2009; Raman et al., 2013; Jing et al., 2013)
- Vehicle power management (Ozay et al., 2011a)
- Camera network control (Ozay et al., 2011b)



#### So is that the end of the story?

#### And everyone lived happily ever after...

# So is that the end of the story?

And everyone lived happily ever after...

...well, not quite. There is also:

- Noise
- Imprecise modelling of the environment
- Incomplete information
- Scalability
- Robustness / Error-resilience
- Quirks of the synthesis algorithm
- ...















# Example for error-resilience



#### Focus of this talk

#### A few answers to how we can deal with...

- Noise
- Imprecise modelling of the environment
- Incomplete information
- Scalability
- Robustness / Error-resilience
- Quirks of the synthesis algorithm

# Synthesizing *error-resilient* implementations

# Example for (discrete) robustness (revisited)



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## Spectrum of robustness (in discrete synthesis)



This figure is certainly not complete and only lists *some* approaches.

#### k-Resilient synthesis (Def.: Huang et al., 2012)



# k-Resilient synthesis (Def.: Huang et al., 2012)



#### Algorithmic approach

Given a GR(1) specification  $\psi$  and k, we can encode the k-resilient synthesis problem of  $\psi$  by translating  $\psi$  to a modified GR(1) specification  $\psi'$  and perform GR(1) synthesis for  $\psi'$ .

# The Reduction from GR(1) to GR(1) in a nutshell

#### A counter for k

- We ask the system to be synthesized to output a **counter** that says how many *glitches* can be tolerated in the near future.
- If glitches stop occurring, then the system must eventually set the counter back to *k*.

#### Effect

- Quantification over b is abstracted by a liveness property
- Idea can be implemented by altering the specification.

# Example: Robot patrolling

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# Example: Robot patrolling



# Example: Robot patrolling



# Synthesizing cooperative controllers

Based on joint work with Roderick Bloem and Robert Könighofer, IROS 2015

#### Demo

								Strategy Visualizer										

#### Main problem

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#### Naive approaches to solving this problem

 Require the system to satisfy the specification even if the environment is *naughty*
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  → defeats the purpose of synthesis

### Our approach

We modify the main generalized reactivity(1) synthesis approach to compute only *cooperative* controllers.

# Making GR(1) synthesis cooperative

# Synthesis objective

- All executions must be correct.
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 Prevent the system from issuing a next output that forces the environment to subsequently violate its specification

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### Implementation: Liveness

• Modify the GR(1) fixpoint computation

# Standard GR(1) fixpoint equation

$$\nu Z. \bigwedge_{j \in \{1,...,n\}} \mu Y. \bigvee_{i \in \{1,...,m\}} \nu X. \operatorname{EnfPre}\left( \left( Z' \wedge \psi_j^g \right) \vee Y' \vee \left( \neg \psi_i^a \wedge X' \right) \right)$$

# Cooperative GR(1) synthesis

$$vZ. \bigwedge_{j \in \{1,...,n\}} \mu Y. \bigvee_{i \in \{1,...,m\}} vX. \text{EnfPre}((Z' \land \psi_j^g) \lor Y' \lor (\neg \psi_i^a \land X'))$$
$$\land \mu R. \text{Reach}((\psi_j^g \lor Y' \lor R') \land X)$$
$$\land \bigwedge_{k \in \{1,...,m\}} \mu R. \text{Reach}((\psi_k^a \lor R') \land Z)$$

# Demo

			Strategy	Visuali:	zer				- + ×

# Optimal control in adversarial environments

Based on joint work with Gangyuan Jing and Hadas Kress-Gazit (published at IROS 2013)

# Adding an optimization criterion

### Basic idea

In addition to the specification, we introduce a cost function.

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Example due to Chatterjee and Henzinger (2006)

# Adding an optimization criterion

# Specification parts

- The door can be open or closed.
- ∀*i* ∈ {1,2,3,4}: If button *i* is pressed, floor *i* is eventually visited with the door open.
- At every step, the current floor number is not increased or decreased by more than one.
- The current floor number can only be changed if the door is closed.
- The "door close" command can fail or succeed, while the "open door" command always succeeds.

### Optimization criterion / cost function

- Every operation has a (fixed) cost.
- We want to minimize the average cost per execution step.

# What is an optimal strategy?

### Optimal strategy (mean-payoff)

- Service requests for 1 step, then wait for 1 step, then
- service requests for 1 step, then wait for 2 steps, then
- service requests for 1 step, then wait for 3 steps, then
- . . .

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### So what now?

We need to fix either:

- specification
- weigths/costs
- optimization objectives

# What is an optimal strategy?

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# Example

r <sub>1</sub>	r <sub>2</sub>	r <sub>3</sub>	r <sub>4</sub>	r <sub>5</sub>	r <sub>6</sub>	<b>r</b> 7
r <sub>8</sub>	rg		<i>r</i> <sub>10</sub>		r <sub>11</sub>	r <sub>12</sub>

# Specification parts

Assumptions:	Guarantees	
<b>GF</b> (open <sub>1</sub> )	$\mathbf{G}(\neg open_1 \rightarrow \mathbf{X}(\neg r_3)))$	
$GF(open_2)$	$\mathbf{G}(\neg open_2 \rightarrow \mathbf{X}(\neg r_5)))$	<b>F</b> <i>r</i> <sub>7</sub>
$GF(open_3)$	$\mathbf{G}(\neg open_3 \rightarrow \mathbf{X}(\neg r_{10})))$	

# Example



### Specification parts

# Introducing "action cost"



# Introducing "action cost"



# Introducing "action cost"



Computing action cost along a path

• Take the sum of costs until reaching the next goal

### Basic idea

Waiting in strategies can be detected by looking at the SCCs.

<i>r</i> 1	r <sub>2</sub>	r <sub>3</sub>	r <sub>4</sub>	<i>r</i> 5	r <sub>6</sub>	<b>r</b> 7
ľ8						ľ12
.0	r <sub>9</sub>		r <sub>10</sub>		r <sub>11</sub>	12

### Basic idea

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### But what about repetitive tasks?

Here, we count SCCs up to the point of reaching the next goal.



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# Conditions on the specification

### Idea

- If the specification has a set of *goals* for the system, we can count the number of waiting cycles for reaching the respective *next* goal.
- After reaching the next goal, the counter resets.
- It is the aim of the system to reach the next goal cheaply.

### Using the idea for GR(1) specifications

- Still singly-exponential complexity
- Strategy shape is the same as for standard GR(1) synthesis: positional-per-goal

# Combining action and waiting cost

# Two-dimensional cost notion

From every state of a strategy, a strategy has a *cost tuple* that describes the cost to reach the next goal.



# Combining action and waiting cost

# Two-dimensional cost notion

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### Example

Cost of path 1 from  $r_1$ :  $(c_w, c_a) = (2, 8)$ , cost of path 2:  $(c_w, c_a) = (1, 14)$ Overall cost: depends on the **preference** 

# Preference relations

### Preference relations

- We use a preference relation ≤<sub>P</sub> to choose which executions the strategy should prefer (e.g., (1, 14) ≤<sub>P</sub> (2, 8)).
- The value of a strategy from some state is the highest combined cost (w.r.t. ≤<sub>P</sub>) of all executions originating from the state.

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# Using the idea for GR(1) specifications

For almost-linear preference relations, we can still compute optimal strategies in exponential time. They track:

- The current atomic proposition values
- The "current" liveness assumption and liveness guarantee
- Whether  $\infty$  action cost can still be avoided

# Risk-Averse Control of Markov Decision Processes with $\omega$ -regular Objectives

Based on joint work with Salar Moarref and Ufuk Topcu (published at CDC 2016)

# Basic problem setup

# **Basic Problem**

 We want to control a Markov decision process (MDP) such that an ω-regular specification is satisfied...



# Basic problem setup

### **Basic Problem**

- We want to control a Markov decision process (MDP) such that an ω-regular specification is satisfied...
- ...but we want to do this in MDPs in which all policies have a probability of 0 for satisfying the specification.



# Example problem



# So how can we control the system?

### Idea

We compute a controller that maximizes the probability to reach the next *goal*.

From every goal (or initially), A *p*-risk averse controller reaches the next goal with probability at least *p*.
### So how can we control the system?

#### Idea

We compute a controller that maximizes the probability to reach the next *goal*.

From every goal (or initially), A *p*-risk averse controller reaches the next goal with probability at least *p*.

#### But what is the next goal?

When working with general  $\omega$ -regular specifications, this is not so easy to tell!



#### Acceptance



#### Acceptance





#### Acceptance





#### Acceptance



# Connecting Deterministic Parity Automata and MDP Control

#### Basic idea

We let the *controller* always tell the *current goal color* and when it just reached a goal.

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The controller may always *increase* the goal color, but *decrease* it only finitely a fixed number of times.

# Connecting Deterministic Parity Automata and MDP Control

### Basic idea

We let the *controller* always tell the *current goal color* and when it just reached a goal.

The controller may always *increase* the goal color, but *decrease* it only finitely a fixed number of times.

#### Finding *p*-risk averse policies

- For every p ∈ [0,...,1], a p-risk averse control policy has a finite number of states
- Optimal strategies can be computed by solving a series of optimal reachability policy computations in MDPs.

# **Estimator-based synthesis**

Based on joint work with Ufuk Topcu, HSCC 2015

### Example application: Distance keeping assistant



Observable: Noisily Measured: Unobserved: Controlled: Speed of the follower car Distance between cars Speed of the leader car Acceleration (follower)

### Synthesis – complexity vs. expressivity (incomplete inf.)



### Synthesis – complexity vs. expressivity (incomplete inf.)



### Synthesis – complexity vs. expressivity (incomplete inf.)



**Central question:** How to retain the singly-exponential complexity of GR(1) synthesis under incomplete information?









# Decoupling estimator computation from synthesis

### Main ideas

- We decouple the estimation of the physical values and reactive synthesis using an estimator specification as glue.
- We modify the controller specification to only talk about observable variables.

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Example specification parts		
Estimator specification	<b>Controller Specification</b>	
$G(minDistance \leq distance)$	$G(minDistance \geq 5)$	
$G(maxDistance \ge distance)$		









### Scalable estimator computation

#### Problem

In general, estimator computation is still a doubly exponential problem

# Scalable estimator computation

### Our solution

We only consider **positional estimators**. These may only base their next estimates on:

- the last sensor values and the last estimates
- the current sensor values
- the possible evolutions of the environment

### Properties of our approach

- fixed size of the estimators
- no "strategic planning" possible by the estimators
- unique optimal estimators exist for most estimate preference relations

# Example: Discretized car following controller (1)

### Properties

- distance  $\in \{0, ..., 84, 85\}$
- speedLeader  $\in \{0, \dots, 15\}$
- speedFollower  $\in \{0, \dots, 15\}$
- accelerationLeader  $\in \{-2, -1, 0, 1, 2\}$
- accelerationFollower  $\in \{-2, -1, 0, 1, 2\}$
- Noisy distance update
- Approximate (±2) distance measurement

• . . .

### Specification parts

- The distance must always be at least 5.
- G(distance < 85  $\lor$  speedFollower = 15)

# Example: Discretized car following controller (2)



Meaning of the encoding :	(observedDistance speedFollower accFollower minSpeedLeader maxSpeedLeader minDist maxDist
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# Example: Discretized car following controller (2)



Meaning of the encoding	observedDistance speedFollower accFollower minSpeedLeader maxSpeedLeader minDist maxDist
----------------------------	--

Example: Discretized car following controller (3)



	observedDistance speedFollower
Meaning of the encoding	accFollower minSpeedLeader maxSpeedLeader minDist maxDist

Computing positional estimators

1. Compute the reachable states of any estimator

$$R = \mu X.(\{x_0\} \cup \{x' \subseteq \mathsf{AP}_{obs} \cup \mathsf{AP}_{hid} \cup \mathsf{AP}_{est} \mid \exists x \in X.$$
$$(x \setminus \mathsf{AP}_{est}, x' \setminus \mathsf{AP}_{est}) \in \rho_e, (x, x') \in \rho_s\})$$

2. Compute which estimates are admissible

$$\begin{split} \rho_{u} = & \{ (x, x') \in (2^{\mathsf{AP}_{obs} \cup \mathsf{AP}_{est}})^{2} \mid \forall y, y' \subseteq 2^{\mathsf{AP}_{hid}} : \\ & ((x \cup y) \in R \land ((x \setminus \mathsf{AP}_{est} \cup y), \\ & (x' \setminus \mathsf{AP}_{est} \cup y'))) \in \rho_{e} \rightarrow ((x \cup y), (x' \cup y')) \in \rho_{s} \} \end{split}$$

### 3. Restriction to optimal estimates

$$\hat{\rho}_{u} = \{ (x, x') \in \rho_{u} : x'|_{\mathsf{AP}_{est}} = \min\{x''|_{\mathsf{AP}_{est}} : (x, x'') \in \rho_{u}, \\ = x' \setminus \mathsf{AP}_{est} = x'' \setminus \mathsf{AP}_{est} \} \}$$

## Discretized car following controller (cont'd)

### Computation times with slugs (BDD-based)

- Basic scenario: 22+28 minutes
- Cruise mode scenario: 22+460 minutes (6 realizability checks)

#### Without estimator-based synthesis

Belief space: 2<sup>16-86</sup> states - beyond tractability

# Conclusion

# "Spicing up CPS controller synthesis"

#### In this talk...

...we discussed a few approaches to make the concept of *reactive synthesis* more applicable to CPS controller computation.

# "Spicing up CPS controller synthesis"

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...we discussed a few approaches to make the concept of *reactive synthesis* more applicable to CPS controller computation.

#### But can they be combined?

Apart from the *error-resilient synthesis* part, they all require modifications of the synthesis process.  $\rightarrow$  **So no!** 

# "Spicing up CPS controller synthesis"

### In this talk...

...we discussed a few approaches to make the concept of *reactive synthesis* more applicable to CPS controller computation.

### But can they be combined?

Apart from the *error-resilient synthesis* part, they all require modifications of the synthesis process.  $\rightarrow$  **So no!** 

#### Ok, so what now?

We will need to research methods to combine the considerations presented in this talk in a larger framework that still allows for scalable synthesis!
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