Symmetric Strategy Improvement
RiSE Seminar, Pöllauberg

Sven Schewe
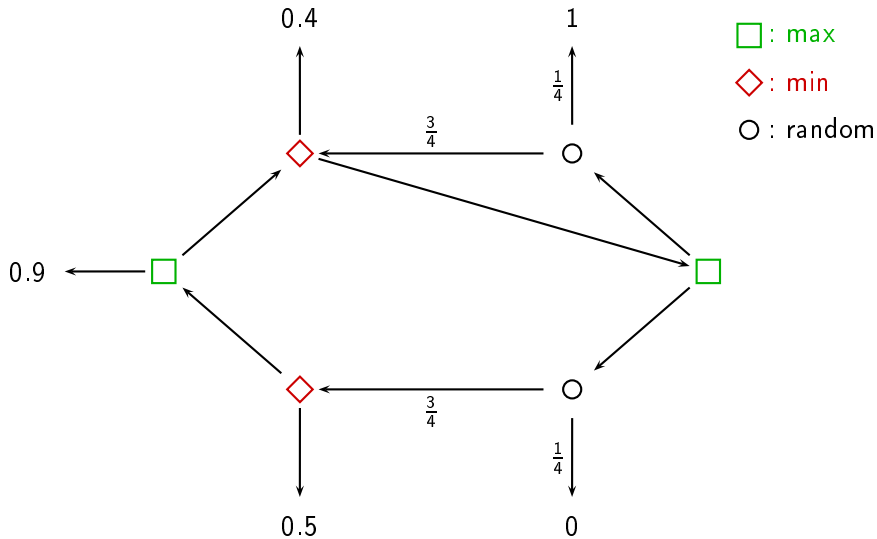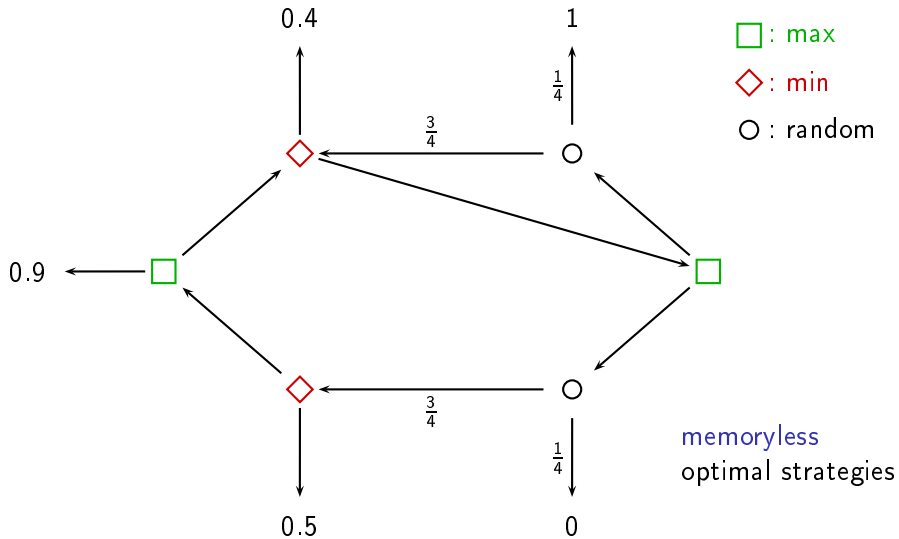
University of Liverpool

September 26th, 2016

Symmetric Strategy Improvement

1. Parity / Payoff / Simple Stochastic Games

2. Why you should **not** listen to this talk

3. Strategy improvement

Simple Example

Simple Example

Applications

- emptiness/acceptance games
  for parity tree automata

- synthesis & satisfiability checking

- quantitative verification

- economic games

Applications

who cares? it's fun!

Don't Listen!

It's Intriguing

It looks so . . .

- simple
- beautiful

- much like our typical playground . . .

Obvious Facts and Open Questions

Obvious Facts

- symmetric
- $\Rightarrow$ in class $\cap$ co-class

- **single** fixed point of DPG can be guessed
- $\Rightarrow$ in UP $\cap$ co-UP   [Jurdziński 98]

Open Problems

- # P?
- RP / ZPP?
- pay-off games: $2^{O(\sqrt{n})}$?, $2^{o(n)}$?

It's Fun

Proposition

It's fun to toy around with these structures.

Proof

Rather obvious and left to the reader.

It's Deceitful
– and not always rewarding –

You always think a PTIME solution is around the corner . . .

It's Deceitful
– and not always rewarding –

... but there is usually only another sign ...

It's Deceitful

– and not always rewarding –

...and another ...

Sexy

1. simple setting
2. defies solution

   pocket version of P vs. NP

   ($\notin P \Rightarrow$ much harder)

What you shouldn't listen to

Strategy Improvement

- check—optimise—check   the classic approach
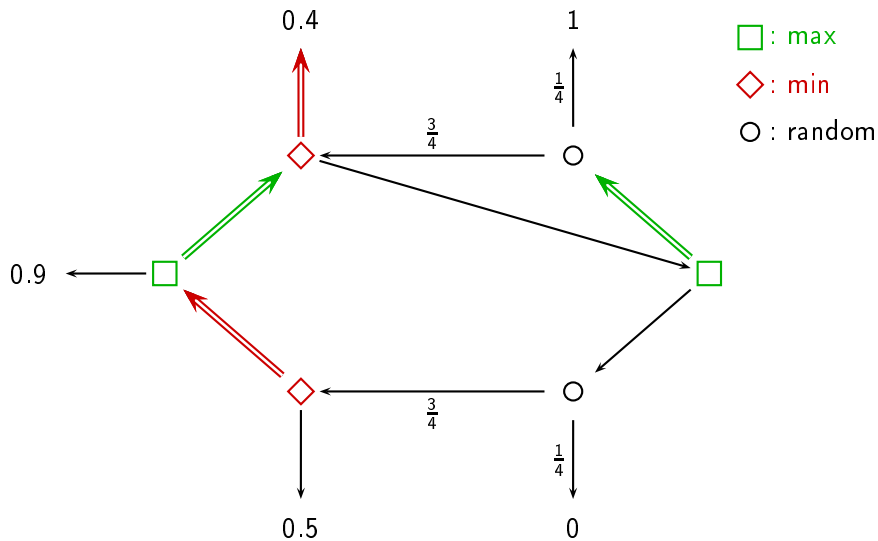   was long-time P hopeful

- why symmetry?
- why not?
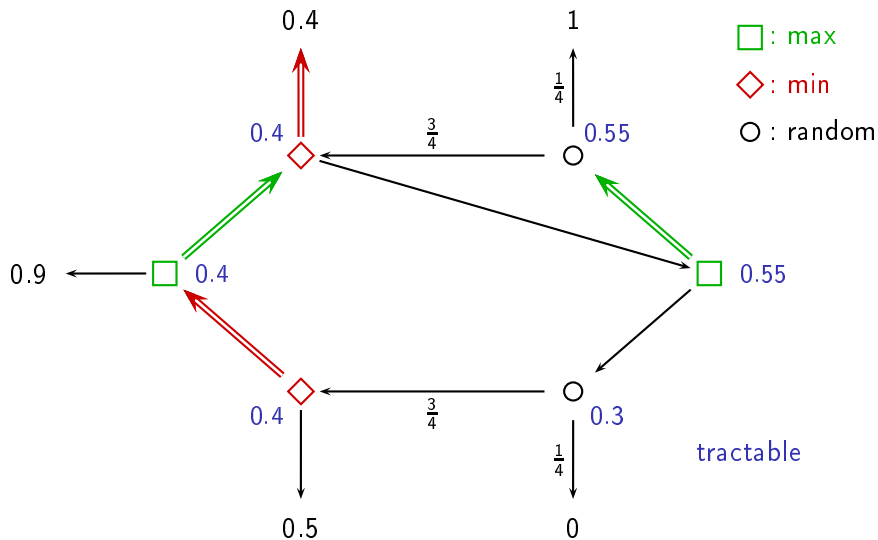
- the approach
- annecdotal evidence

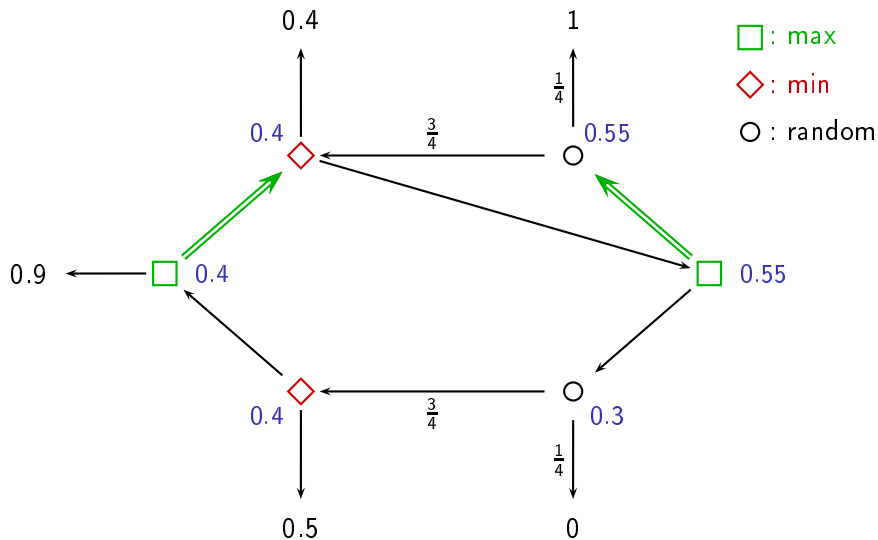# Strategy Improvement

Classic Strategy Improvement

fix strategy

Classic Strategy Improvement

find best response and . . .
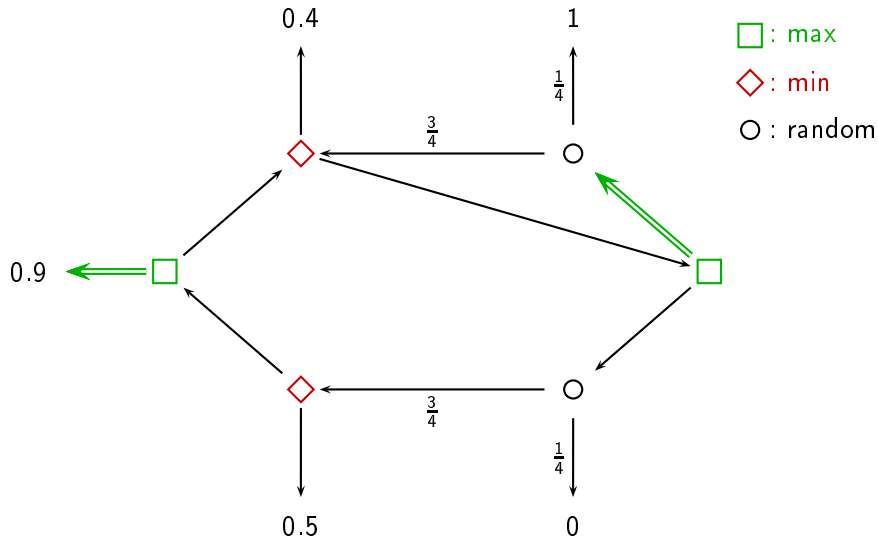
Classic Strategy Improvement

... evaluate

Classic Strategy Improvement
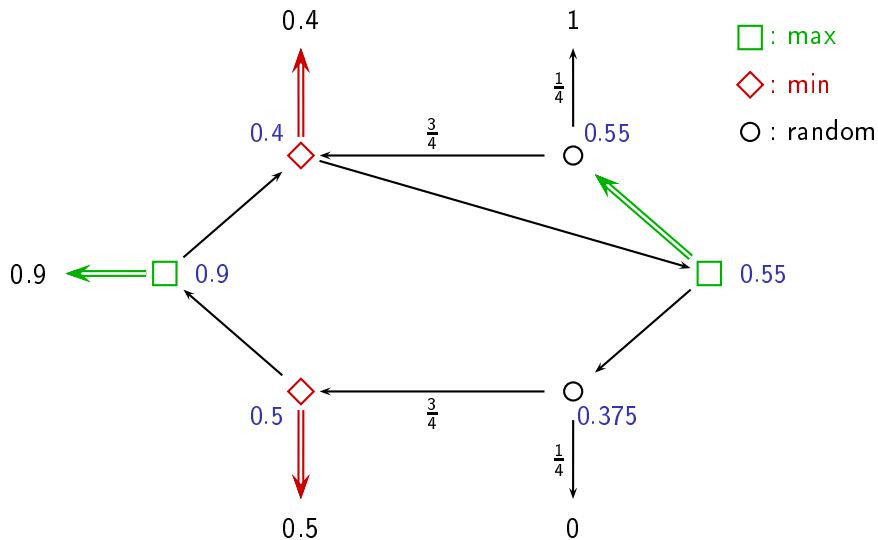
apply local improvements

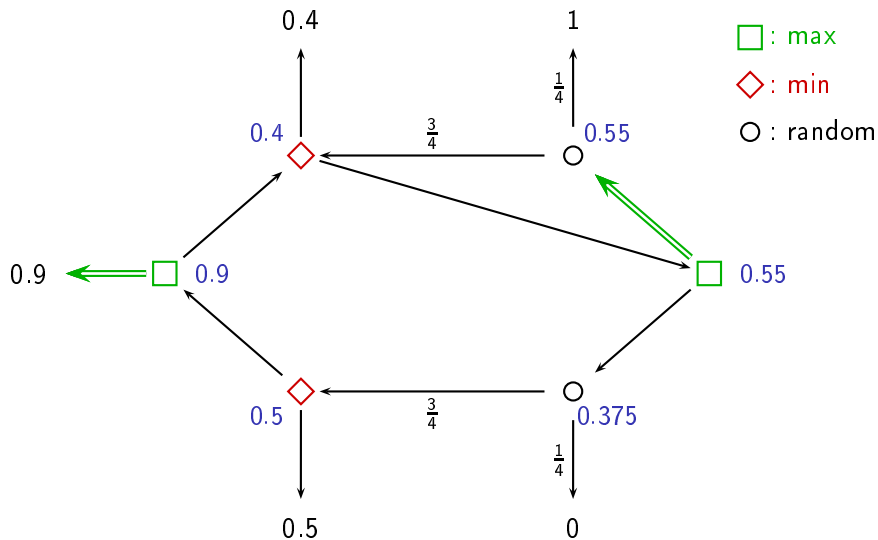# Classic Strategy Improvement
apply local improvements

Classic Strategy Improvement
find best response & evaluate

0.4

1

0.4

$\frac{3}{4}$

0.55

$\square$: max

$\diamond$: min

○ : random

0.9

0.9

0.55

0.5

$\frac{3}{4}$

0.375

$\frac{1}{4}$

0.5

0

Classic Strategy Improvement
no local improvent: done

□ : max

◇ : min

○ : random

CSI – failed hope

- was long hoped to be tractable
- many update policies

$\forall$ exponential lower bounds   [Friedmann 11,...]
   – use static update policy

$\exists$ PSPACE powerful   [Fearnley+Savani 15]

SYMMETRYЯTƎMMYS

Symmetry and Complexity   [Jurdziński 98]

1. guess valuation
2. verify
⇒ one value: UP
   symmetry: UP∩CoUP

Iterated Fixed Point [Emerson+Lei 86]   parity games

- similar treatment
- best performing algorithm

Optimal Strategy Improvement   [Schewe 08]
                        parity games, MPG mean partitions

- some symmetry
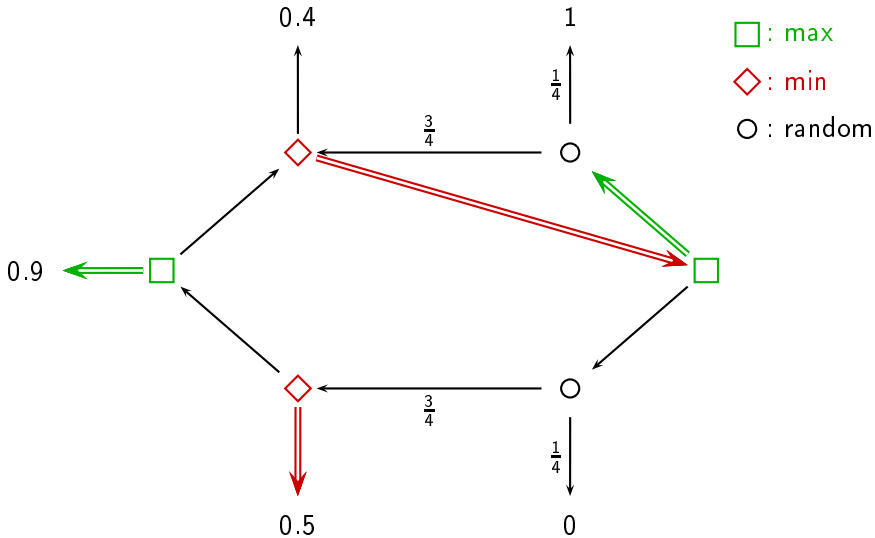- fab performance

Why not?

> **Naive symmetric strategy improvement**
>
>   Question: Why has SSI not been thoroughly studied?
>   Answer: Anne Condon has proved it wrong   [Condon 93]
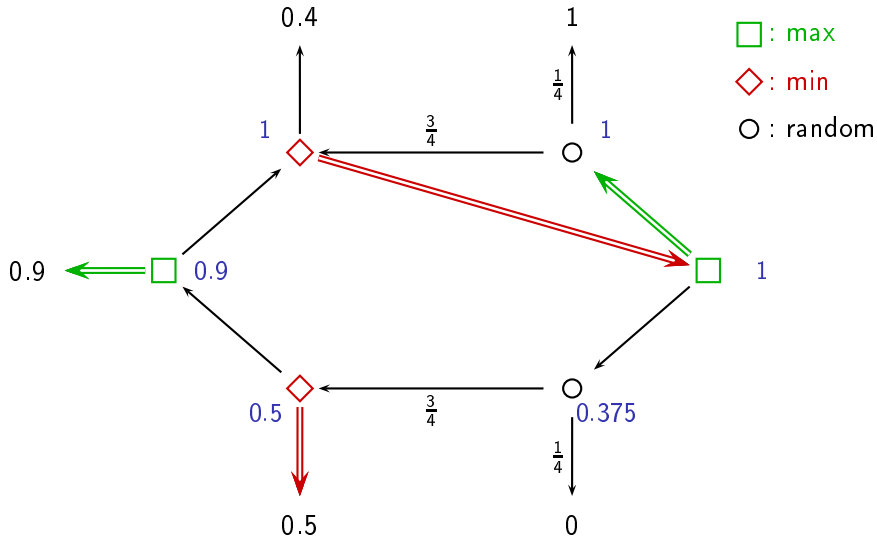
1. Cuncurrent Switch
2. Alternating Best Response

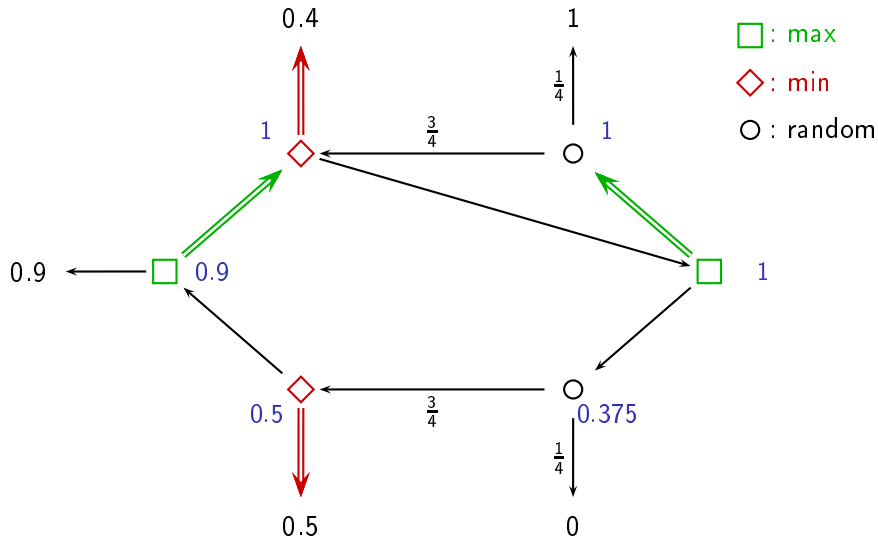# Concurrent Switch
starting strategies

Concurrent Switch
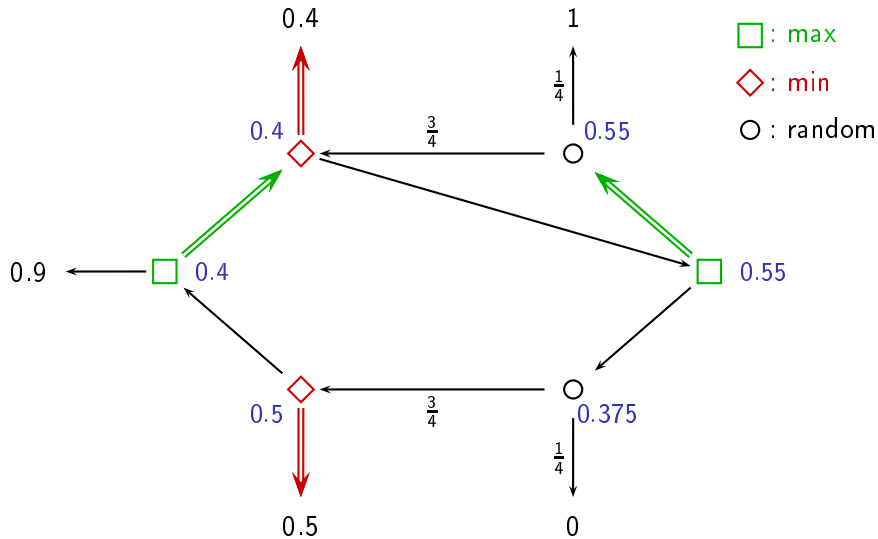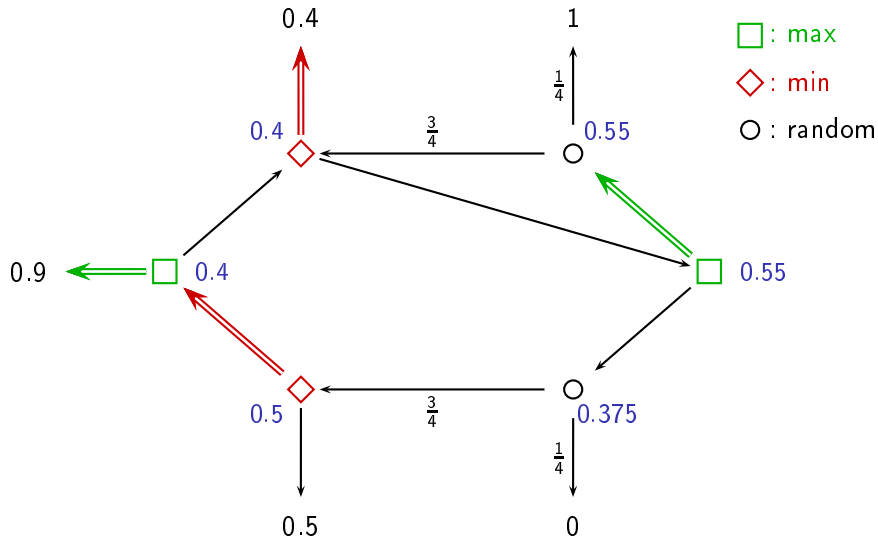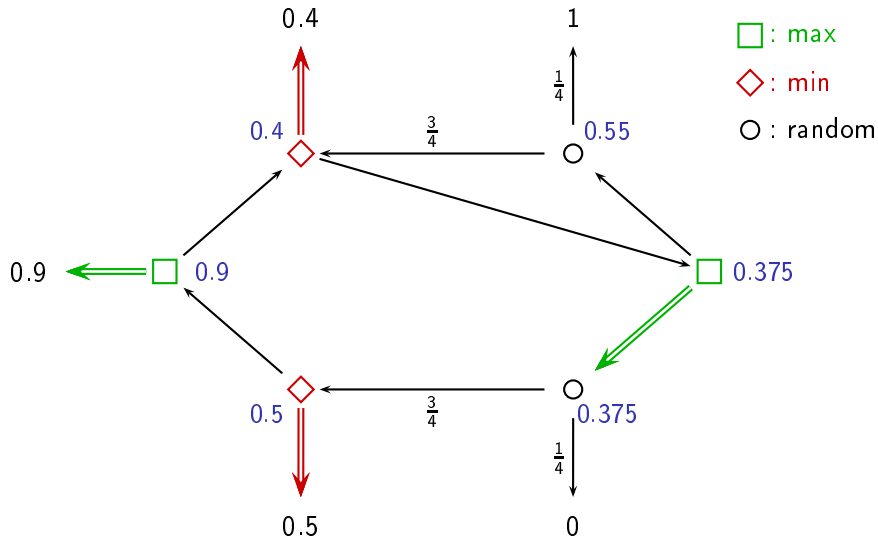
evaluate

Concurrent Switch

update strategies

Concurrent Switch

update evaluation

Concurrent Switch

update strategies

$\square$: max

$\diamondsuit$: min

$\bigcirc$: random

Concurrent Switch

update evaluation

$\square$: max

$\diamond$: min

$\bigcirc$: random
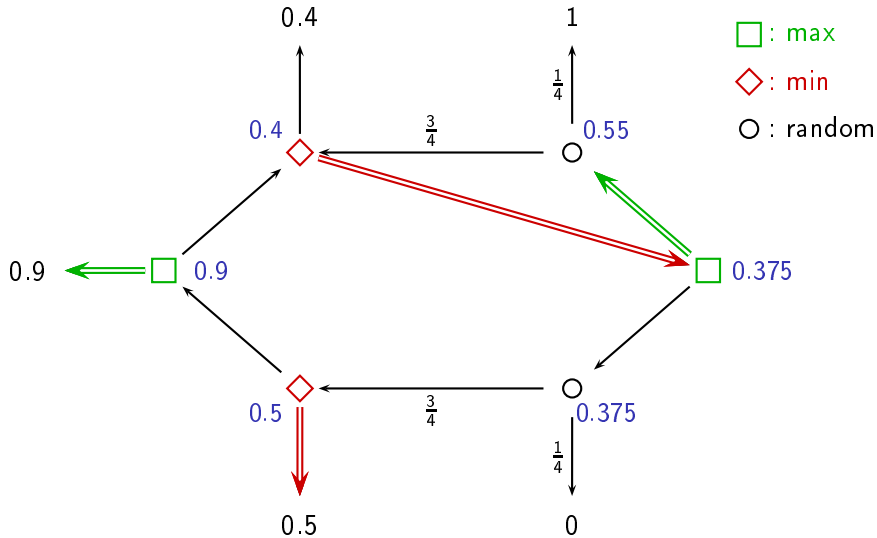
Concurrent Switch
update strategy
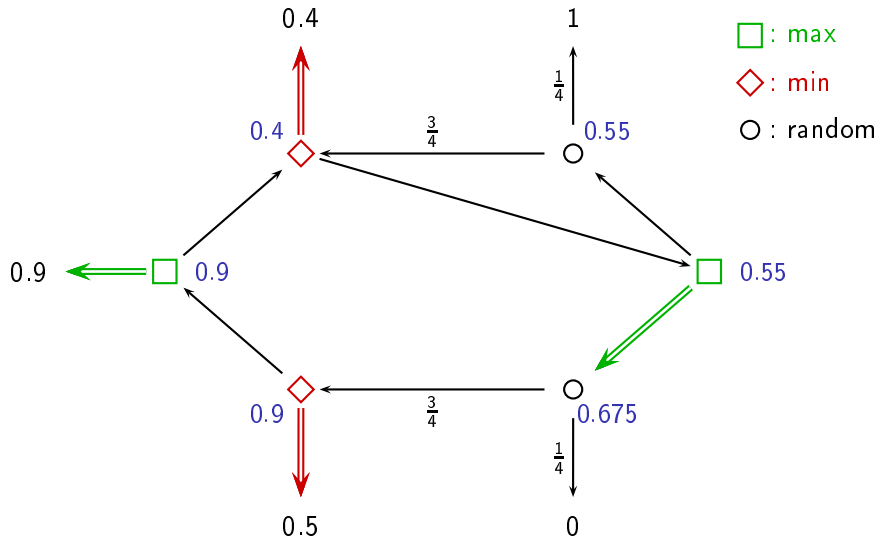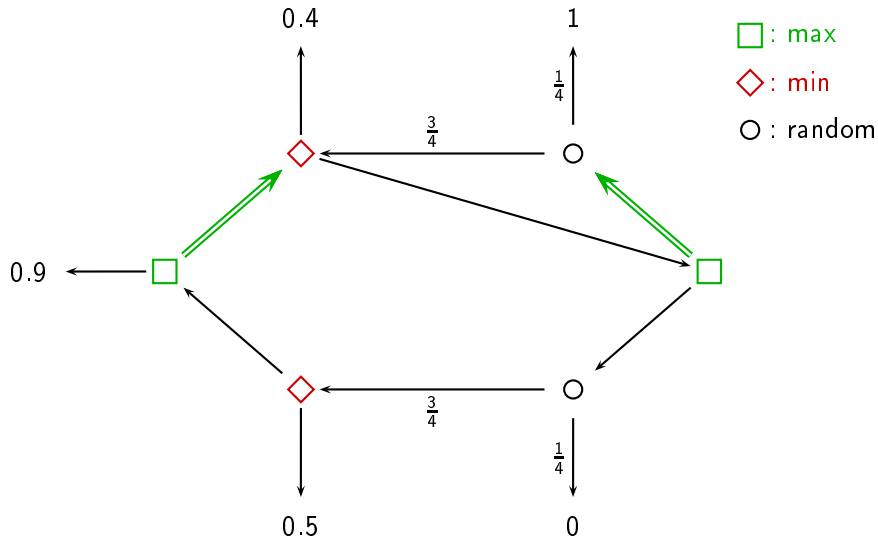
Concurrent Switch

update evaluation

Concurrent Switch
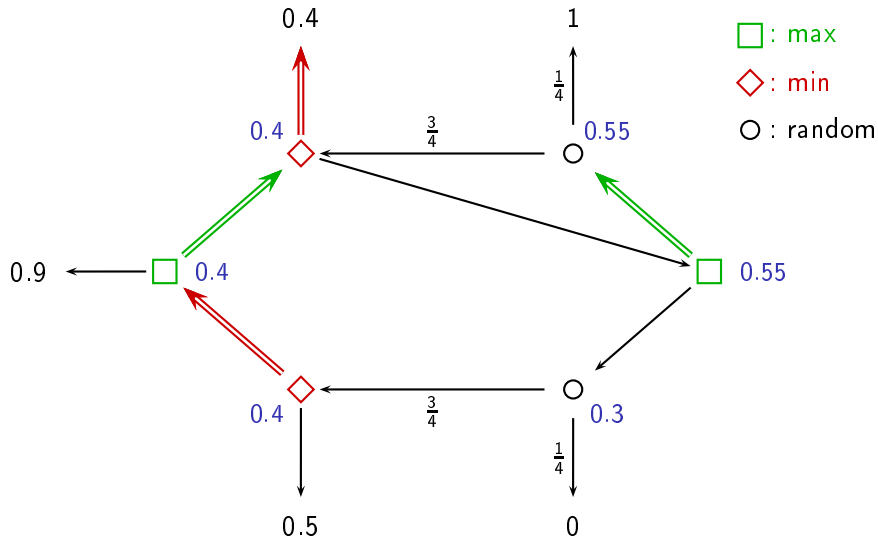## update strategy (cycle)

Alternating Best Response

strategy

Alternating Best Response

best response

Alternating Best Response

strategy

Alternating Best Response

best response

Alternating Best Response

strategy

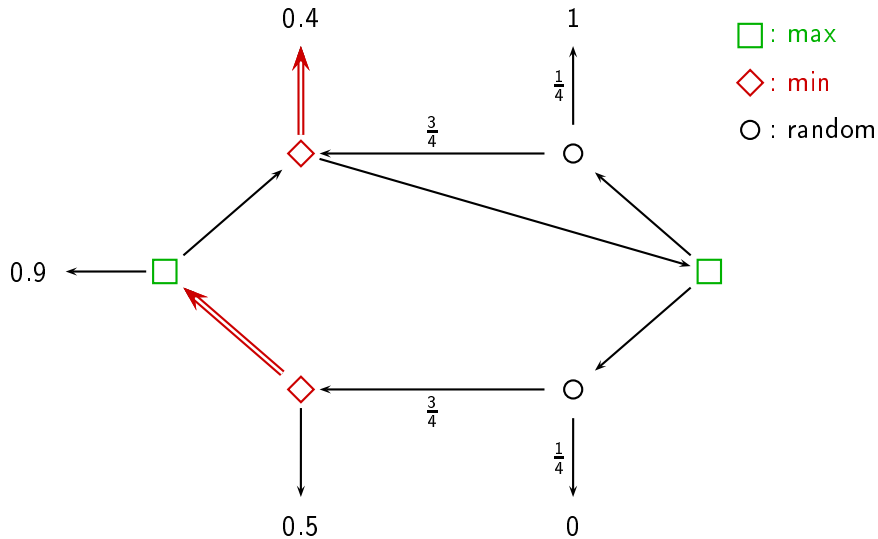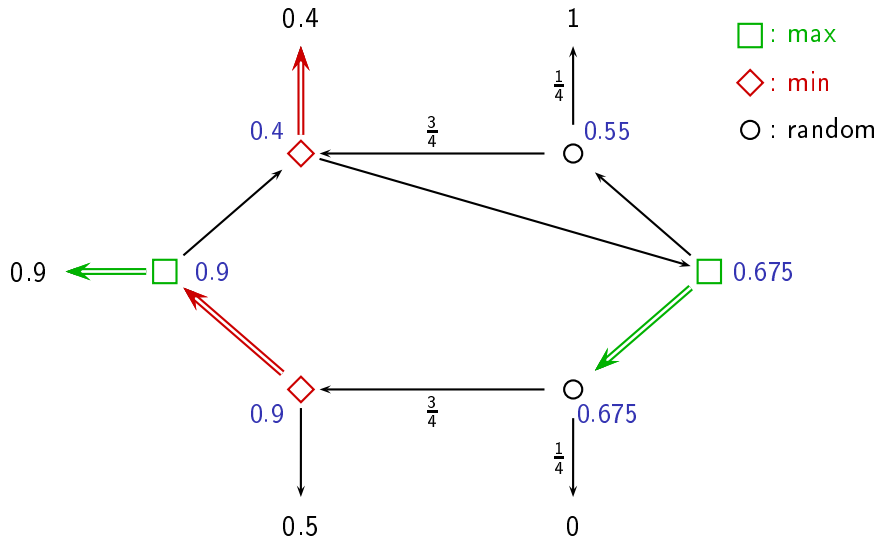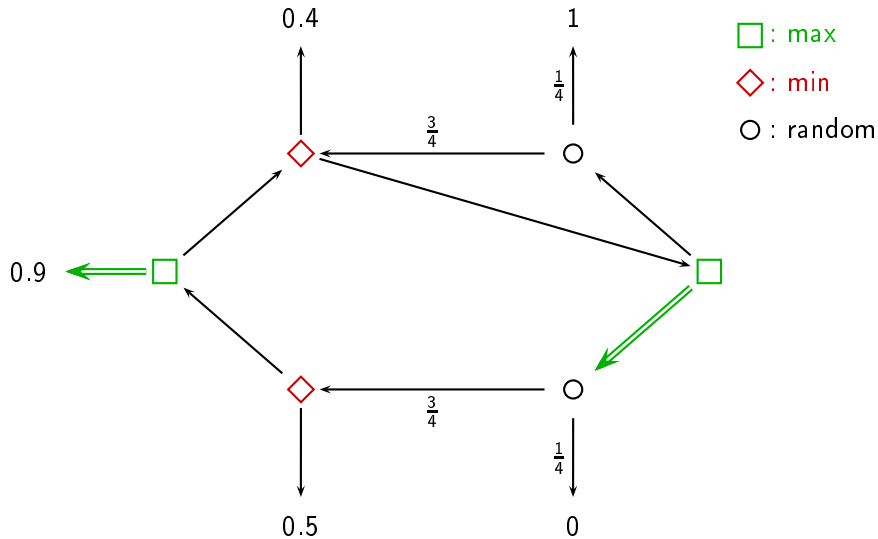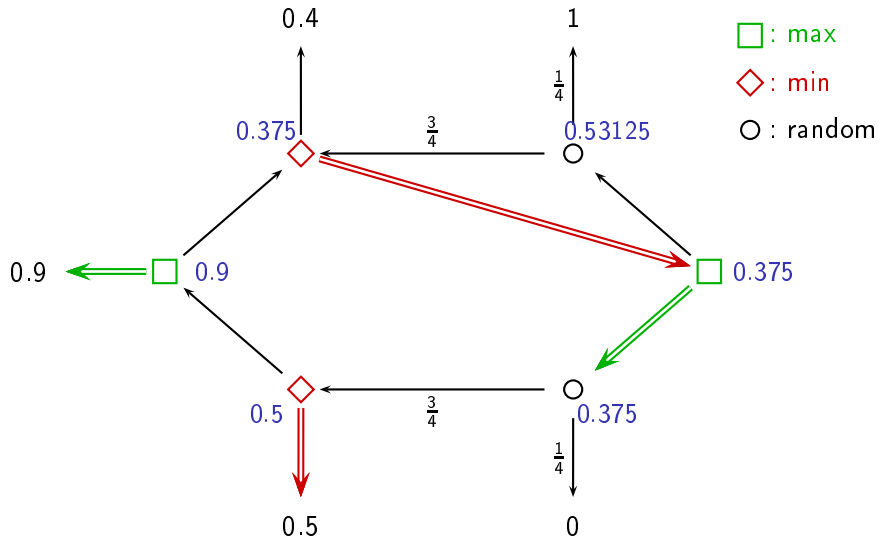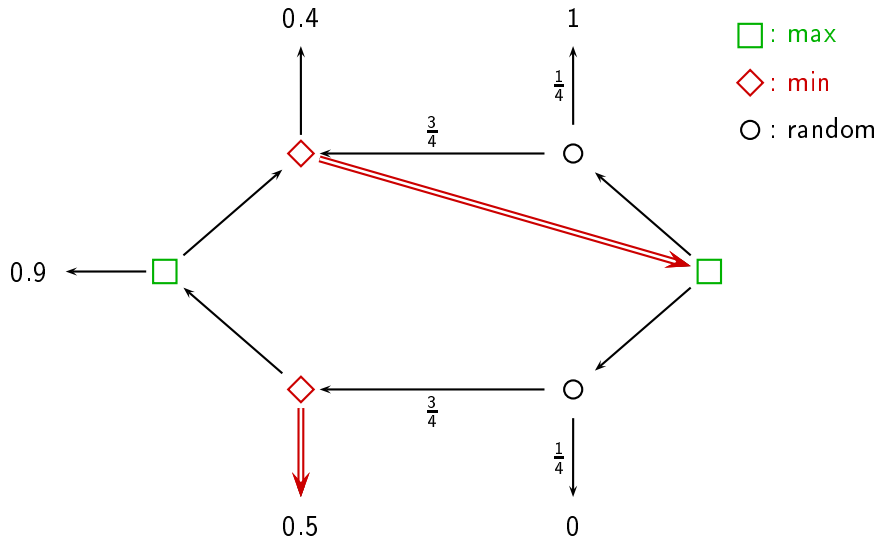# Alternating Best Response

best response

Alternating Best Response

strategy

Alternating Best Response

best response (cycle)
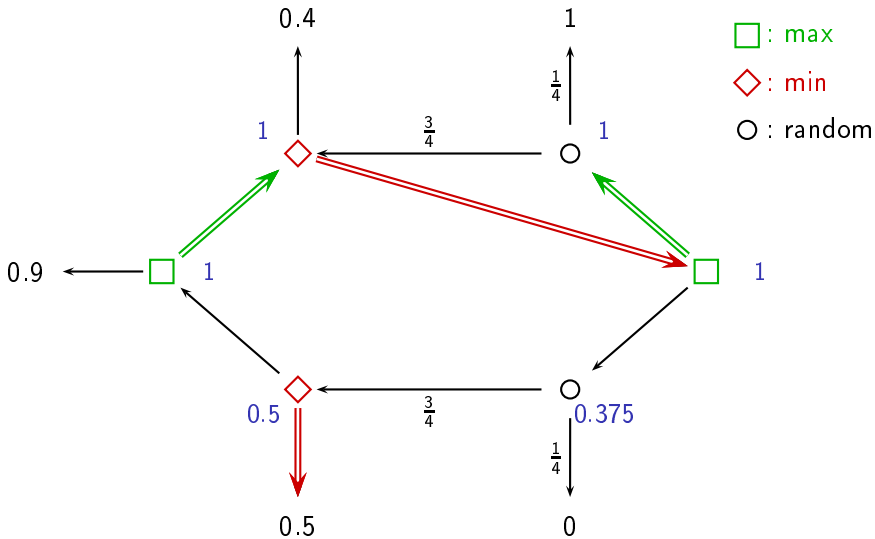
Symmetric Strategy Improvement

Symmetric Strategy Improvement
starting strategies

□ : max
◇ : min
○ : random
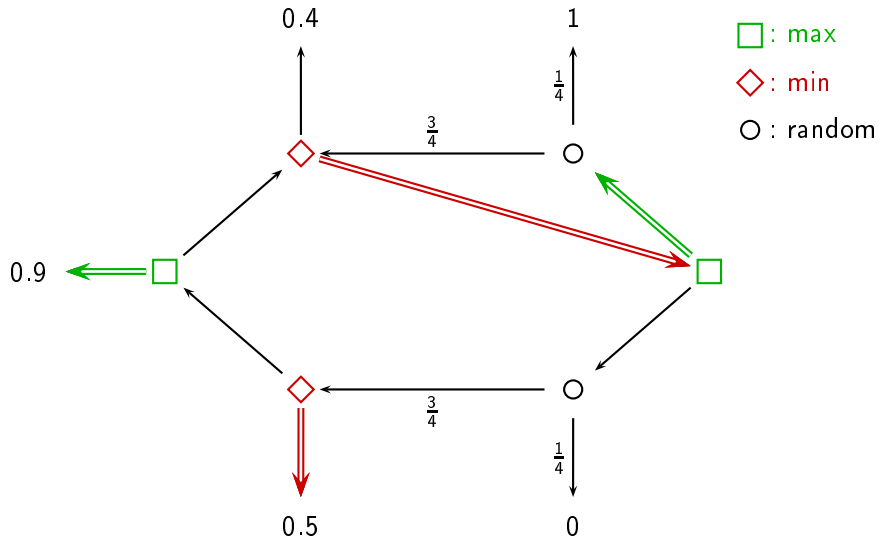
Symmetric Strategy Improvement
evaluate
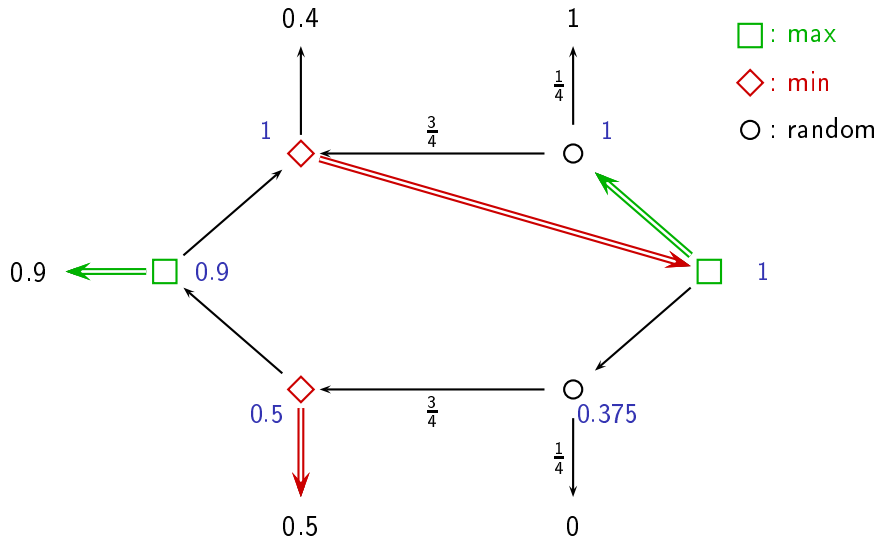
Symmetric Strategy Improvement

best response

Symmetric Strategy Improvement
best response & improvement

$\square$: max

$\diamond$: min

$\bigcirc$: random
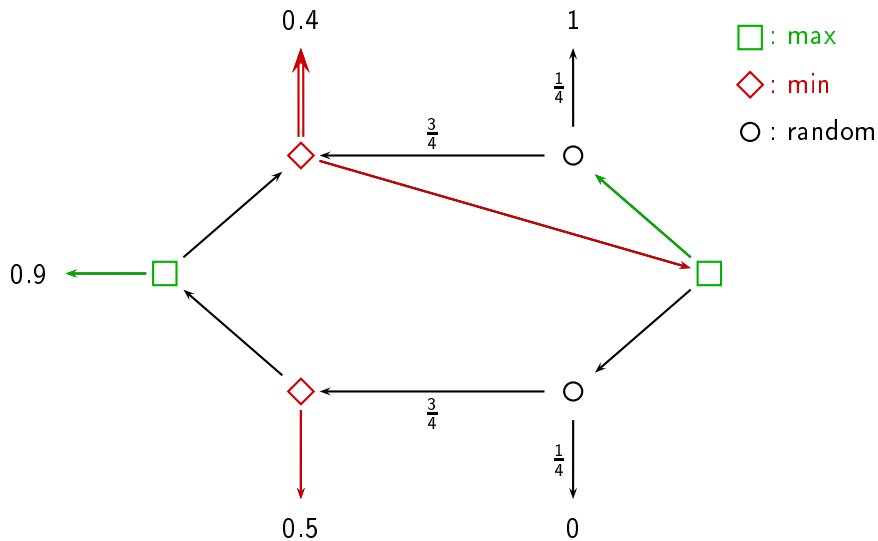
Symmetric Strategy Improvement

update (done)

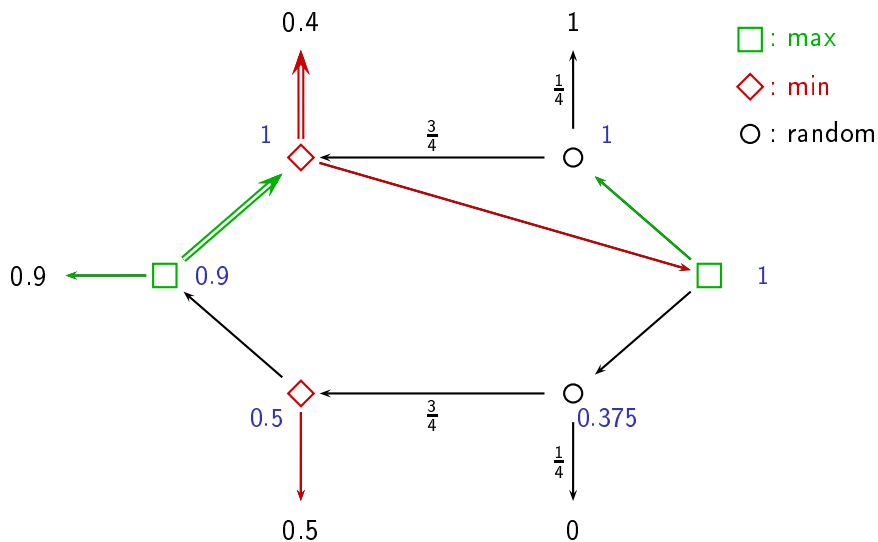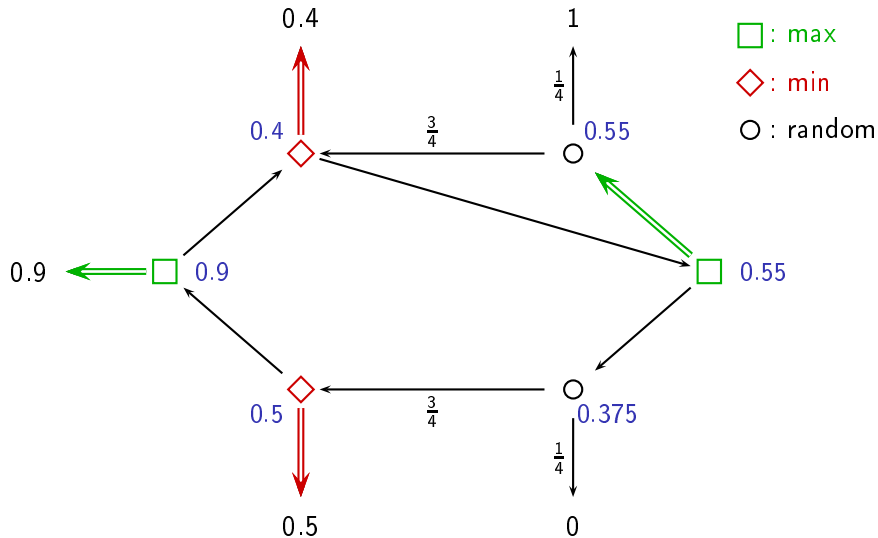# Symmetric Strategy Improvement

Can SSI help overcome problems of CSI?

Question: How about single player examples?   [Fearnley 10]

Answer: Easy (but no surprise there)

Question: How about Friedmann's traps?   [Friedmann 11,...]

Answer: Yes but this doesn't imply there are no traps

Question: Less iterations on random games?

Answer: Yes but probably not half

Question: Is SSI polynomial?

Answer: Look at the weather! Isn't it lovely?

Friedmann's Traps

Friedmann's Traps

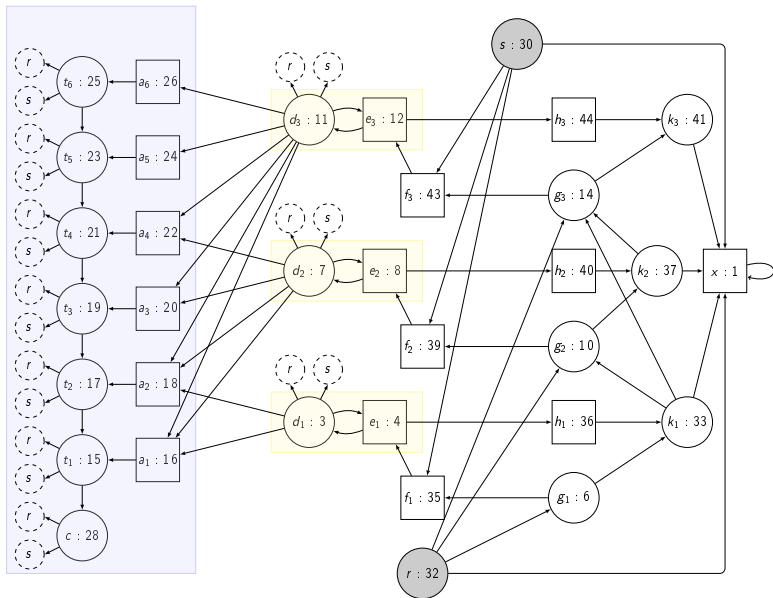| Switch Rule	1	2	3	4	5	6	7	8	9	10
Cunningham	2	6	9	12	15	18	21	24	27	30
CunninghamSubexp	1	1	1	1	1	1	1	1	1	1
FearnleySubexp	4	7	11	13	17	21	25	29	33	37
FriedmannSubexp	4	9	13	15	19	23	27	31	35	39
RandomEdgeExpTest	1	2	2	2	2	2	2	2	2	2
RandomFacetSubexp	1	2	7	9	11	13	15	17	19	21
SwitchAllBestExp	4	5	8	11	12	13	15	17	18	19
SwitchAllBestSubExp	5	7	9	11	13	15	17	19	21	23
SwitchAllSubExp	3	5	7	9	10	11	12	13	14	15
SwitchAllExp	3	4	6	8	10	11	12	14	16	18
ZadehExp	-	6	10	14	18	21	25	28	32	35
ZadehSubexp	5	9	13	16	20	23	27	30	34	37

SYMMETRYRYTƎMMYS

beautiful
google **'symmetry beauty'** to find out how well this is researched!

natural
- one of the first algorithmic ideas studied
- folklore: doesn't work

mysterious   complexity
- defies intractability proofs
- but are there others?

tempting   future work
unveil the secret